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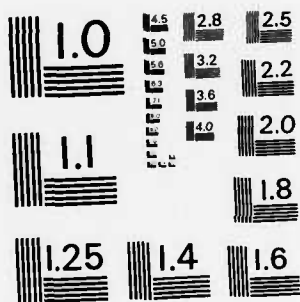
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ENVIRONMENTAL IMPACT RESEARCH PROGRAM

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TWO-EQUATION, DEPTH-INTEGRATED TURBULENCE CLOSURE FOR MODELING GEOMETRY-DOMINATED FLOWS

by

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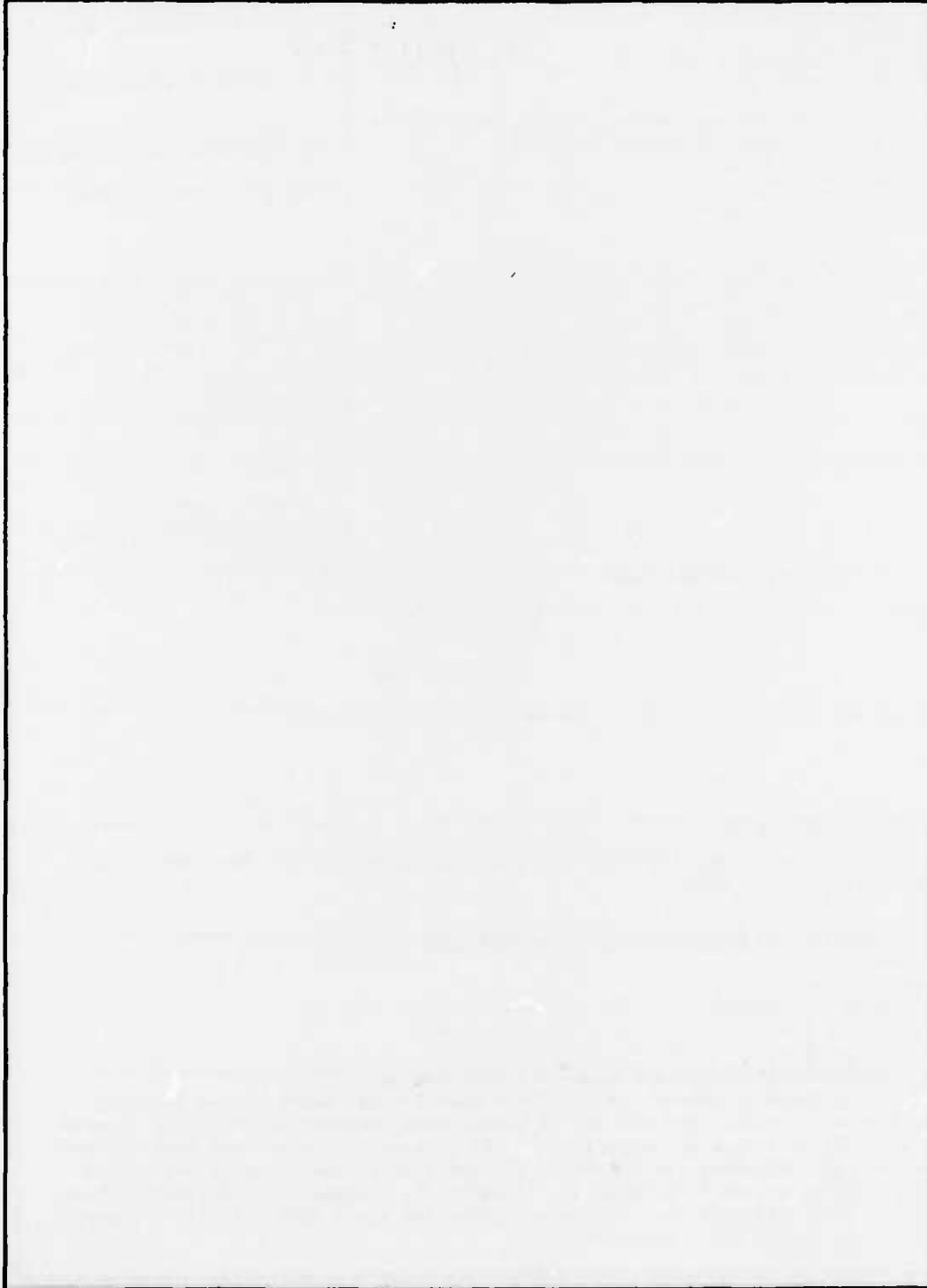
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Preface

This report was prepared by Dr. Raymond S. Chapman for the U. S. Army Engineer Waterways Experiment Station (WES), Vicksburg, Miss. During report preparation, Dr. Chapman was Research Associate, Virginia Polytechnic Institute and State University, Blacksburg, Va., on contract to WES through Intergovernmental Personnel Act Assignment Agreement, IPA-80-19. The study was sponsored by the Office, Chief of Engineers (OCE), under the Environmental Impact Research Program (EIRP). Mr. John Bushman was OCE Technical Monitor. Dr. Roger Saucier was EIRP Program Manager.

The study was monitored at WES by Mr. Ross Hall, Environmental Research and Simulation Division (ERSD), Environmental Laboratory (EL). Mr. Donald Robey, Chief, ERSD, and Dr. John Harrison, Chief, EL, provided general supervision.

Director of WES during report preparation was COL Tilford C. Creel, CE. Technical Director was Mr. F. R. Brown.

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TWO-EQUATION, DEPTH-INTEGRATED TURBULENCE CLOSURE
FOR MODELING GEOMETRY-DOMINATED FLOWS

Introduction

1. When applying depth-integrated hydrodynamic and dispersion models to investigate large-scale water quality problems in coastal environments, it is usually adequate to adopt the simplest of eddy viscosity/diffusivity closure hypotheses in which the depth-integrated eddy viscosity/diffusivity coefficient is either assumed to be constant or related to the water depth and shear velocity. However, when local geometry-dependent flow phenomena such as separation and recirculation are deemed important to the water quality investigation, it is then necessary to adopt a more sophisticated level of closure for the depth-integrated equations of motion and constituent transport equations.

2. It is the intent of this discussion to (a) describe the closure problem associated with the use of the depth-integrated equations of motion, (b) provide a detailed description of the two-equation ($k-\epsilon$) turbulence closure model, and (c) present the results of steady-state test simulations utilizing $k-\epsilon$ turbulence closure.

3. In addition, the limitations of the model developed by the author are discussed with specific recommendations for future improvement.

Turbulence Closure

Depth-integrated equations
of motion and closure problem

4. Under the assumption of a homogeneous, incompressible, viscous flow characterized by a hydrostatic pressure distribution, with wind and Coriolis forces neglected, the depth-integrated equations of motion are written:

Conservation of Mass

$$\frac{\partial h}{\partial t} + \frac{\partial (V_m h)}{\partial X_m} = 0 \quad (1)$$

Conservation of Momentum

$$\frac{\partial (V_m h)}{\partial t} + \frac{\partial (V_m V_n h)}{\partial X_n} + g \frac{\partial (h^2/2)}{\partial X_m} + gh \frac{\partial z_b}{\partial X_m} + \tau_{bm} - \frac{\partial T_{mn}}{\partial X_n} = 0 \quad (2)$$

where

$m, n = 1, 2$, and repeated indices require summation

t = time

V_m = two-dimensional depth-averaged velocity vector (U, V)

h = water depth

X_m = coordinate directions (x, y)

g = acceleration due to gravity

τ_{bm} = components of the bottom shear stress per unit mass

z_b = channel bottom elevation above an arbitrary datum

T_{mn} = components of the depth-integrated effective stress tensor per unit mass

5. The depth-integrated effective shear stress tensor as defined by Kuipers and Vreugdenhil (1973) and Flokstra (1977) contains the (I) viscous stresses, (II) the turbulent Reynold's stresses, and (III) the momentum dispersion terms which arise from depth integrating the nonlinear convective acceleration terms in the equations of motion. Specifically, the depth-integrated effective stress tensor per unit mass is written:

$$T_{mn} = \int_{z_b}^{h+z_b} \left[\underbrace{v \left(\frac{\partial v_m}{\partial X_n} + \frac{\partial v_n}{\partial X_m} \right)}_{(I)} - \underbrace{\overline{v'_m v'_n}}_{(II)} - \underbrace{(v_m - \overline{v_m})(v_n - \overline{v_n})}_{(III)} \right] dz \quad (3)$$

where

ν = kinematic viscosity

\bar{v}_m = time-averaged velocity components (u , v)

v'_m = horizontal turbulent velocity fluctuations

6. The closure problem associated with the use of the depth-integrated equations of motion results from the need to parameterize terms II and III in the effective stress tensor.

7. The contribution of the viscous stresses (term I) can be neglected simply because its effects are transparent to the computation at the scales of motion modeled in a fully turbulent hydraulic problem.

8. In the recent literature, the treatment of Reynolds stress closure (term II) has ranged from neglecting the terms to the concept of "large eddy" simulations (Leonard 1974) where, by means of spatial filtering of the equations of motions, only the small-scale or "subgrid-scale" Reynolds stresses need be modeled. Within these two extremes lie a number of alternative closure schemes which exhibit a wide variation in complexity (Reynolds 1976).

9. One closure technique that has enjoyed considerable success in the simulation of a variety of turbulent flows is the two equation (k - ϵ) turbulence model described by Launder and Spalding (1974) and Rodi (1980). Applications of a depth-integrated version of the k - ϵ turbulence model have been presented by Rastogi and Rodi (1978) and McQuirk and Rodi (1978). A detailed discussion of the three-dimensional and depth-integrated version of the k - ϵ closure model is the subject of the next section.

10. Parameterization of the momentum dispersion terms (III) is straightforward if one has an a priori knowledge of the vertical distribution of the horizontal velocity components. Unfortunately, theoretical velocity distributions are available for only the simplest of flows such as flow in a wide channel or long circular channel bend. Discussions of the importance of the momentum dispersion terms and approximate closure schemes are presented by Flokstra (1977), Abbott and Rasmussen (1977), and Lean and Weare (1979). A common feature of all of the existing closure schemes for momentum dispersion is that the magnitude of the components of momentum dispersion that can be important is

directly proportional to the ratio of the depth of flow to the radius of curvature of the depth-mean streamlines. Consequently, if one restricts their attention to flows that are much wider than deep, which is usually the case in estuarine and other coastal flows, then the need to address momentum dispersion is obviated.

Depth-averaged turbulent
Reynold's stress closure

11. The k - ϵ turbulence closure model presented by Launder and Spalding (1974) is based on the Boussinesq eddy viscosity hypothesis (Hinze 1959), which assumes that the turbulent Reynold's stresses are proportional to the mean strain rates. Using three-dimensional tensor notation, the turbulent Reynold's stresses are written:

$$\overline{-v'_i v'_j} = -\frac{2}{3} k \delta_{ij} + \nu_t \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \quad (4)$$

where

$i, j = 1, 2, 3$, and repeated indices require summation

$k = \frac{1}{2} \overline{v'_i v'_i}$, the turbulent kinetic energy per unit mass

δ_{ij} = Kronecker delta

Unlike the molecular viscosity ν , the turbulent eddy viscosity ν_t is flow-dependent and can vary both in space and time. An approximation for the distribution of the turbulent eddy viscosity is obtained by assuming that it is proportional to the product of the characteristic velocity and length scale of turbulence, namely:

$$\nu_t \propto k^{1/2} \ell \quad (5)$$

where ℓ equals the macroscale of turbulence (a measure of the size of the energy containing eddies). An inviscid estimate of the energy dissipation rate per unit mass ϵ is obtained when one assumes that the amount of energy dissipated at the small scales of turbulence equals the rate of supply at the large scales.

12. Again utilizing the characteristic velocity and length scales of turbulence, dimensional considerations require that (Tennekes and Lumley 1972)

$$\epsilon \sim \frac{k^{3/2}}{l} \quad (6)$$

Substitution of Equation 6 into Equation 5 yields a functional relationship for the turbulent eddy viscosity in terms of the kinetic energy of turbulence k , and its rate of dissipation ϵ , specifically:

$$\nu_t = C_v \frac{k^2}{\epsilon} \quad (7)$$

where C_v is an empirical coefficient.

13. In principle, solution of the exact transport equations for the turbulence kinetic energy k , and its rate of dissipation ϵ , enables one to completely specify the temporal and spatial distribution of the turbulent eddy viscosity. However, construction of the transport equations for k and ϵ results in an additional closure problem. Consider first the turbulence kinetic energy equation (Hinze 1959):

$$\frac{\partial k}{\partial t} + \frac{\partial (kv_i)}{\partial x_i} = \frac{\partial}{\partial x_i} \left[\overline{v_i' \left(\frac{P'}{\rho} + k \right)} \right] - \overline{(v_i' v_j')} \frac{\partial v_j}{\partial x_i} \quad (8)$$

(I)
(II)

$$+ \nu \frac{\partial}{\partial x_i} \left[\overline{v_j' \left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i} \right)} \right] - \overline{\nu \left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i} \right) \frac{\partial v_j'}{\partial x_i}} \quad (IV)$$

(III)

where P' denotes turbulent pressure fluctuations, ρ is the fluid density, and the over-bar represents a time average. The closure problem results from the presence of the unknown pressure and velocity fluctuation correlations in term I. Physically, term I represents the

convective diffusion of the total turbulence mechanical energy per unit mass by turbulence. This term acts as a redistribution mechanism, which suggests the use of a gradient diffusion model (Rodi 1980), or:

$$\overline{v'_i \left(\frac{P'}{\rho} + k \right)} = \frac{v_t}{\sigma_k} \frac{\partial k}{\partial x_i} \quad (9)$$

where σ_k is an empirical constant. Equation 4 may be substituted directly for the turbulent Reynold's stresses in the turbulence production term (II). Term IV is by definition the energy dissipation rate per unit mass, ϵ . Finally, term III represents the work done by the viscous shear stresses. For high Reynold's number flows, this term is small and can be neglected (Hanjalic and Launder 1972). Making the appropriate substitutions, the three-dimensional model equation for the turbulence kinetic energy is written:

$$\frac{\partial k}{\partial t} + \frac{\partial (v_i k)}{\partial x_i} = \frac{\partial}{\partial x_j} \left(\frac{v_t}{\sigma_k} \frac{\partial k}{\partial x_j} \right) + v_t \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \frac{\partial v_i}{\partial x_j} - \epsilon \quad (10)$$

14. The exact transport equation for the energy dissipation rate per unit mass, for large Reynold's numbers, reads (Harlow and Nakayama 1968):

$$\frac{\partial \epsilon}{\partial t} + \frac{\partial}{\partial x_i} (\epsilon v_i) = -2\nu \frac{\partial v_i}{\partial x_r} \left(\frac{\partial v'_i}{\partial x_s} \frac{\partial v'_r}{\partial x_s} + \frac{\partial v'_s}{\partial x_i} \frac{\partial v'_r}{\partial x_r} \right) \quad (I)$$

$$- 2\nu \left(\frac{\partial v'_i}{\partial x_r} \frac{\partial v'_i}{\partial x_s} \frac{\partial v'_r}{\partial x_s} \right) - 2 \left(\nu \frac{\partial^2 v'_i}{\partial x_r \partial x_s} \right)^2 \quad (II) \quad (III) \quad (11)$$

$$- \frac{\partial}{\partial x_j} (\overline{v'_j \epsilon'}) - \frac{\nu}{\rho} \frac{\partial}{\partial x_i} \left(\frac{\partial P'}{\partial x_s} \frac{\partial v'_i}{\partial x_s} \right) \quad (IV) \quad (V)$$

where

$r, s = 1, 2, 3$, and repeated indices require summation

ϵ' = turbulent fluctuations of the energy dissipation rate per unit mass

15. The closure approximations for Equation 11 were first presented by Hanjalic and Launder (1972). Their approach was to parameterize I-III in terms of the Reynold's stresses, mean strain rate, turbulence kinetic energy, and its rate of dissipation per unit mass, and to neglect term V on the basis of being small. Term I represents the production mechanisms for ϵ and was approximated accordingly

$$\text{Term I} = -C_1 \left(\frac{\epsilon}{k} \right) \overline{(v'_1 v'_r)} \frac{\partial v_1}{\partial x_r} \quad (12)$$

Terms II and III were grouped together and parameterized as follows:

$$\text{II} + \text{III} = C_2 \frac{\epsilon^2}{k} \quad (13)$$

16. The argument used to support Equation 13 was that the sum of term II, which represents the generation rate of vorticity fluctuations due to the self-stretching mechanism, and term III, which represents the viscous decay of dissipation, should be controlled by the dynamics of the energy cascade. Subsequently, if the Reynold's number is sufficiently large to allow the existence of an inertial subrange, an inviscid estimate based on dimensional considerations should be appropriate. Term IV represents the turbulent diffusion of ϵ , which clearly suggests a closure of the form:

$$\text{Term IV} = \frac{v_t}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial x_j} \quad (14)$$

where σ_ϵ is an empirical constant. Collecting the various approximations yields the complete model equation for the energy dissipation rate per unit mass:

$$\frac{\partial \epsilon}{\partial t} + \frac{\partial (v_i \epsilon)}{\partial x_i} = \frac{\partial}{\partial x_j} \left(\frac{v_t}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial x_j} \right) + C_1 v_t \frac{\epsilon}{k} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \frac{\partial v_i}{\partial x_j} - \frac{C_2 \epsilon^2}{k} \quad (15)$$

17. Estimates for the empirical constants found in Equations 10 and 15 were obtained by applying the model equations to simple turbulent flows for which experimental data were available. For example, in a local equilibrium, two-dimensional boundary layer, the production of turbulence energy is balanced by dissipation, and Equation 10 reduces to:

$$\overline{u'v'} \frac{\partial u}{\partial y} = \epsilon \quad (16)$$

A consistent closure approximation for the Reynold's stress is:

$$\overline{u'v'} = v_t \frac{\partial u}{\partial y} \quad (17)$$

which yields

$$\overline{u'v'}^2 = v_t \epsilon \quad (18)$$

Now, by definition,

$$v_t = C_v \frac{k^2}{\epsilon} \quad (19)$$

Thus, substitution of Equation 18 into Equation 19 results in

$$C_v^{1/2} = \frac{\overline{u'v'}}{k} \quad (20)$$

18. Experimental data extracted from the work of Laufer (1951) suggest that $\overline{u'v'}/k$ varies from 0.22 to 0.3, which gives a range of C_v from 0.05 to 0.09. The value of C_2 was found to lie between 1.9 and 2.0 (Hanjalic and Launder 1972; Launder and Spalding 1974) when

computed using measured decay rates of the turbulent kinetic energy behind a grid (Townsend 1956). The constant C_1 was obtained by examining the form of the dissipation equation in the constant shear stress region near the wall. Here, convection is negligible with production and dissipation approximately in balance, thus

$$\epsilon = \frac{U_*^3}{\kappa y} \quad (21)$$

where

U_* = shear velocity

κ = von Kamans constant

y = distance from the wall

Substituting Equation 21 into Equation 15 with some simplification yields

$$-\frac{v_t}{\sigma_\epsilon} = (C_1 - C_2) \frac{U_*^3 y}{\kappa k} \quad (22)$$

In the near wall region (Townsend 1956)

$$v_t = \kappa U_* y \quad (23)$$

thus

$$C_1 = C_2 - \frac{\kappa^2 k}{\sigma_\epsilon U_*^2} \quad (24)$$

where by definition

$$\frac{U_*^2}{k} = C_v^{1/2} \quad (25)$$

and therefore

$$C_1 = C_2 - \frac{\kappa^2}{\sigma_\epsilon C_v^{1/2}} \quad (26)$$

which specifies the value of C_1 when C_v , C_2 , σ_k , and σ_ϵ are known. The constants σ_k and σ_ϵ , which are similar to turbulent Prandtl or Schmidt Numbers, were assumed to be of order unity and determined along with the recommended values of the other constants via computer optimization. This was done by adjusting the values of the various constants until reasonable agreement between computed and experimental results was obtained. The values recommended by Launder and Spalding (1974) are as follows:

$$C_v = 0.09$$

$$C_1 = 1.44$$

$$C_2 = 1.92$$

$$\sigma_k = 1.0$$

$$\sigma_\epsilon = 1.3$$

With the appropriate specifications of boundary and initial conditions, Equations 7, 10, and 15 and the Launder and Spalding values presented above constitute the complete three-dimensional k - ϵ turbulence closure model.

19. To be of use in approximating the depth-averaged Reynold's stress in Equation 7, it is necessary to cast the three-dimensional k - ϵ model into a depth-integrated form. Realizing that turbulence is inherently three-dimensional, depth integration of the transport equations for the turbulence kinetic energy and its rate of dissipation cannot be strictly performed. However, Rastogi and Rodi (1978) suggest model equations for the depth-averaged turbulence energy \hat{k} , and its rate of dissipation $\hat{\epsilon}$, can, in fact, be constructed if additional source terms are added to account for mechanisms originating from non-uniformity of the flow over the vertical dimension. Furthermore, they

suggest that the resulting turbulent viscosity \hat{v}_t should be interpreted such that when multiplied by the depth-averaged strain rate will yield the depth-averaged turbulent Reynold's stress. By analogy, the depth-averaged Reynold's stress tensor Σ_{mn} is written

$$\Sigma_{mn} = \hat{v}_t \left[\frac{\partial(V_m h)}{\partial X_n} + \frac{\partial(V_n h)}{\partial X_m} \right] - \frac{2}{3} \hat{k} h \delta_{mn} \quad (27)$$

where

$$\hat{v}_t = C_v \frac{\hat{k}^2}{\hat{\epsilon}} \quad (28)$$

20. The resulting model equations for the depth-averaged value of the turbulence energy \hat{k} and its rate of dissipation $\hat{\epsilon}$ are written as follows:

$$\frac{\partial(\hat{k}h)}{\partial t} + \frac{\partial(V_m \hat{k}h)}{\partial X_m} = \frac{\partial}{\partial X_m} \left[\hat{v}_t \frac{\partial(\hat{k}h)}{\partial X_m} \right] + P_h + P_k - \hat{\epsilon}h \quad (29)$$

and

$$\frac{\partial(\hat{\epsilon}h)}{\partial t} + \frac{\partial(V_m \hat{\epsilon}h)}{\partial X_m} = \frac{\partial}{\partial X_m} \left[\frac{\hat{v}_t}{\sigma_\epsilon} \frac{\partial(\hat{\epsilon}h)}{\partial X_m} \right] + \frac{\hat{\epsilon}}{\hat{k}} (C_1 P_h - C_2 \hat{\epsilon}h) + P_\epsilon \quad (30)$$

where

$$P_h = \frac{\hat{v}_t}{h} \left[\frac{\partial(V_m h)}{\partial X_n} + \frac{\partial(V_n h)}{\partial X_m} \right] \frac{\partial(V_m h)}{\partial X_n} \quad (31)$$

21. The source terms P_k and P_ϵ account for the production mechanism resulting from the presence of a vertical boundary layer. The form of these production terms may be obtained by considering the

central portion of a unidirectional, uniform flow in a wide open channel. For this flow, the balance equations for \hat{k} and $\hat{\epsilon}$ reduce to

$$P_k = \hat{\epsilon} h \quad (32)$$

and

$$P_\epsilon = C_2 \frac{\hat{\epsilon}^2 h}{\hat{k}} \quad (33)$$

To a good approximation, the total turbulence energy production over the vertical is written (Townsend 1956):

$$P_k = U_*^2 U \quad (34)$$

but, by definition,

$$U_*^2 = c U^2 \quad (35)$$

where c is a nondimensional friction coefficient; therefore,

$$P_k = c U^3 \quad (36)$$

A similar relation may be obtained for the dissipation source P_ϵ by recalling that from Equation 28

$$\hat{k} = \left(\frac{\hat{v}_t \hat{\epsilon}}{C_v} \right)^{1/2} \quad (37)$$

which may be substituted into Equation 33 to yield

$$P_\epsilon = \frac{C_2 C_v^{1/2} \hat{\epsilon}^{3/2} h}{\hat{v}_t^{1/2}} \quad (38)$$

Noting that

$$\hat{\epsilon} = \frac{U_*^2 U}{h} \quad (39)$$

and introducing the nondimensional dispersion coefficient D :

$$D = \frac{\hat{v}_t}{h U_*} \quad (40)$$

Equation 33 is rewritten:

$$P_\epsilon = \frac{C_2 C_v^{1/2} c^{5/4} U^4}{h D^{1/2}} \quad (41)$$

22. Generalizing these results to two dimensions is simply a matter of replacing the unidirectional flow velocity U with the magnitude of the resultant two-dimensional velocity vector q , specifically:

$$P_k = c q^3 \quad (42)$$

and

$$P_\epsilon = \frac{C_2 C_v^{1/2} c^{5/4} q^4}{h D^{1/2}} \quad (43)$$

23. The interesting feature of this formulation is the introduction of the nondimensional dispersion coefficient in Equation 41, which allows one to specify the value of the free stream turbulent eddy viscosity. Unfortunately, the value of D can vary over two orders of magnitude depending on the geometry of the channel, and, in particular, how one interprets the mechanisms it represents. For example, if one interprets D as representing a vertical turbulent mixing coefficient, Elder (1959) shows that it assumes a value of about 0.07. However, if

one defines D to be a longitudinal dispersion coefficient for an infinitely wide open channel, its value is approximately 5.9. Between these two extremes, an entire spectrum of values may be obtained if D is considered to be a transverse mixing coefficient. Depending on the cross-sectional shape and the longitudinal curvature of the channel investigated, a compilation of numerous experiments yields values ranging from 0.1 to 1.0 (Fischer et al. 1979). Nonetheless, computational experience has shown (Chapman 1982) that a value of unity for the non-dimensional dispersion coefficient yields satisfactory results in the simulation of a depth-integrated wall boundary layer.

Results of Steady-State Test Simulations

24. The model problem chosen for the test simulations was flow in a wide, shallow, rectangular channel with an abrupt, symmetric expansion in width (Figure 1). The reasons for adopting this test problem are

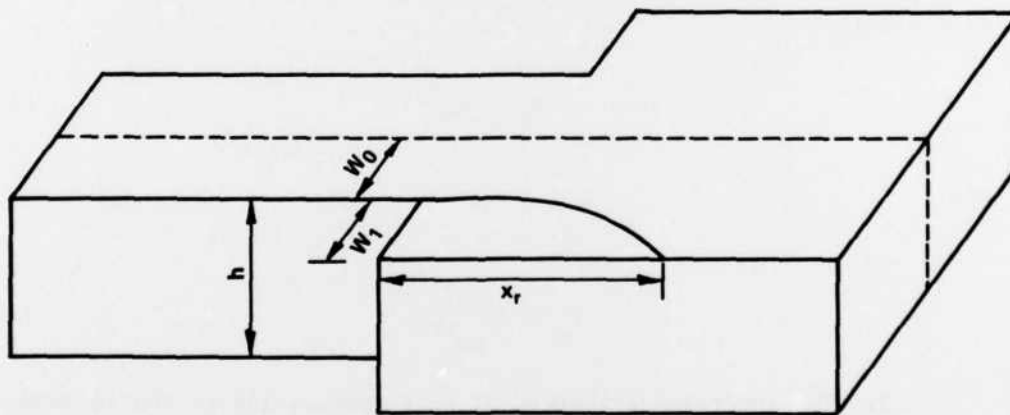


Figure 1. Three-dimensional definition sketch for a channel expansion

essentially twofold. First, a range of reliable nondimensional reattachment lengths x_r/W_1 are available in the form of review papers (Kim, Kline, and Johnston 1978; Durst and Tropea 1981; Eaton and Johnston, 1981). Although the value of the measured nondimensional reattachment varies considerably from one experiment to the next, it was first

pointed out by de Brederode and Bradshaw (1972) that much of the variation in reattachment length measurements could be attributed in differences in the aspect ratios of the test sections. The observation of de Brederode and Bradshaw is well illustrated in Figure 2, a plot of

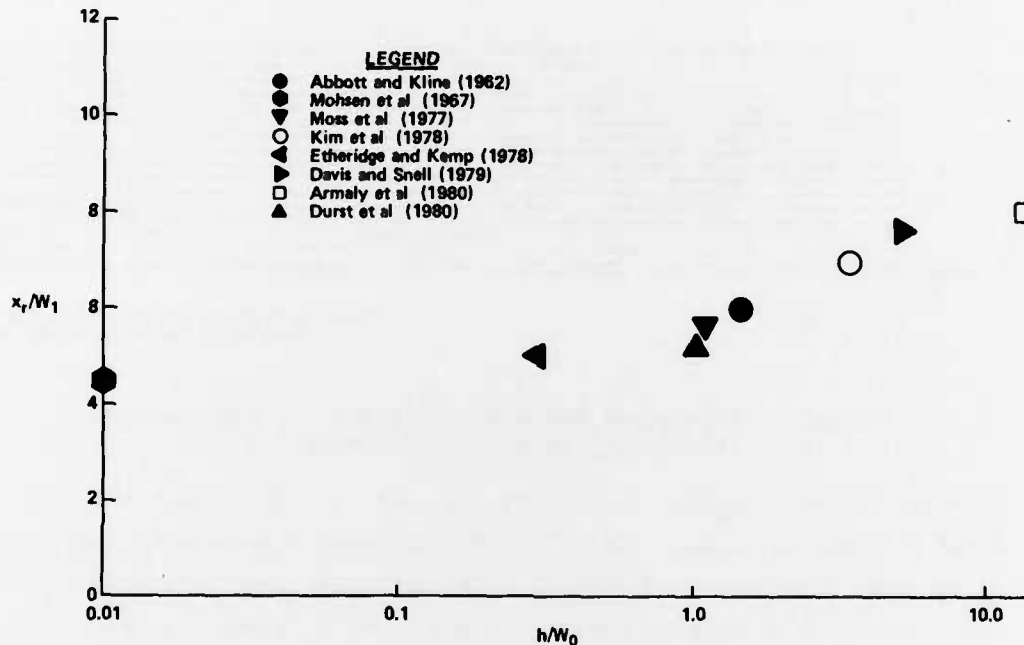


Figure 2. Experimental measurements of nondimensional reattachment lengths versus the inlet channel aspect ratio

published measurements of reattachment lengths for symmetric channel expansions as a function of the upstream section aspect ratio, h/W_0 . In this figure the trend of decreasing reattachment length with decreasing aspect ratio is clearly seen.

25. The second reason for choosing the channel expansion test problem is that previous attempts at simulating this flow, using simple eddy viscosity models, predicted reattachment lengths that were three to four times too short (Abbott and Rasmussen 1977; Ponce and Yabusaki 1981).

26. The details of the test simulations are not presented herein;

however, they may be found in Chapman (1982) or Chapman and Kuo (1982). The results of the initial simulation applying the standard depth-integrated $k-\epsilon$ turbulence closure model, as described in the previous section, are presented in Figure 3, a vector plot of the depth-integrated

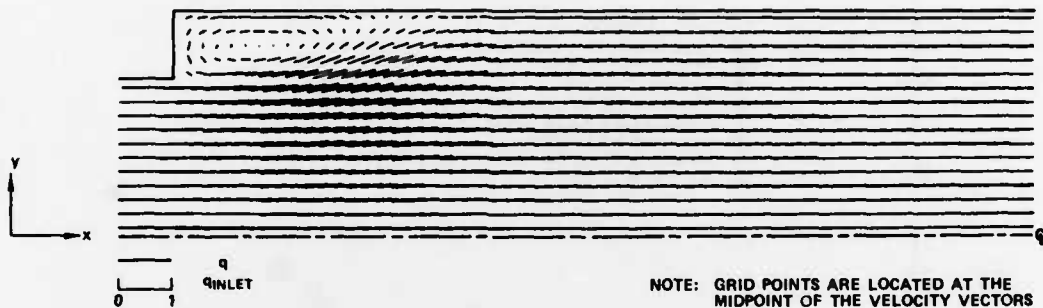


Figure 3. Depth-averaged velocity field for the standard $(k-\epsilon)$ turbulence closure simulation

velocity field. In this simulation, the inlet section aspect ratio is about 0.1, which suggests that the nondimensional reattachment length x_r/W_1 should be about 4.5 to 5.0. However, it is seen in Figure 2 that the predicted reattachment length is only about 3.2, which corresponds to an error of about 30 percent. The poor agreement between model prediction and experimental measurements is directly attributable to dependence on the coefficient C_y to the degree of curvature of the depth-mean streamlines (Leschziner and Rodi 1981). In an attempt to improve upon the model predictions, an ad hoc approximation to the streamline curvature modification of Leschziner and Rodi (1981) was employed. Specifically, the value of the coefficient C_y was decreased to 0.03 at all grid points in the region behind the step for a distance of $6W_1$ downstream.

27. The results of the curvature-corrected simulation are presented in Figure 4 in which a significant increase in the length of the predicted reattachment length is seen. The value of the predicted reattachment length is about $x_r/W_1 = 4.6$, which agrees well with the experimental measurements depicted in Figure 2.

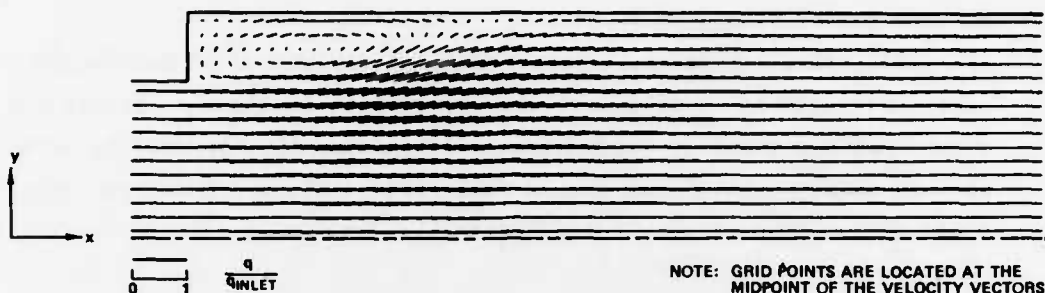


Figure 4. Depth-averaged velocity field for curvature-corrected (k- ϵ) turbulence closure simulation

Model Limitations and Recommendations for Future Work

28. The failure of the standard depth-integrated k- ϵ turbulence closure formulation to predict correct reattachment lengths, and the marked improvement that was realized using an ad hoc streamline curvature correction, suggests that the complete streamline correction formulation of Leschziner and Rodi should be incorporated into the model.

29. Secondly, the present model uses a simple explicit temporal uptake procedure to iterate the solution to steady-state. In order to apply the model to slowly varying environmental flows (i.e., tidal flows), it will be necessary to implement a version in which the water surface elevation is computed implicitly, which removes the overly restrictive gravity wave propagation speed stability criteria (Johnson 1980).

30. Thirdly, if one desires to address rapidly varying transient phenomena, then the existing spatially third-order numerical technique QUICK (Quadratic Upstream Interpolation for Convective Kinematics) of Leonard (1979) must be modified in such a way that it will also be temporally third order. The resulting algorithm, which is called QUICKEST (Quadratic Upstream Interpolation for Convective Kinematics with Estimated Streaming Terms), was presented in one dimension by its

originator Leonard (1979) and extended to a general two-dimensional form by Hall and Chapman (1982).

31. Finally, at the present time, the locations of boundaries are programmed into the mainline of the code, which requires reprogramming with every new geometry configuration. Consequently, it will be necessary to incorporate a flag system that will enable any boundary configuration to be generated via input data.

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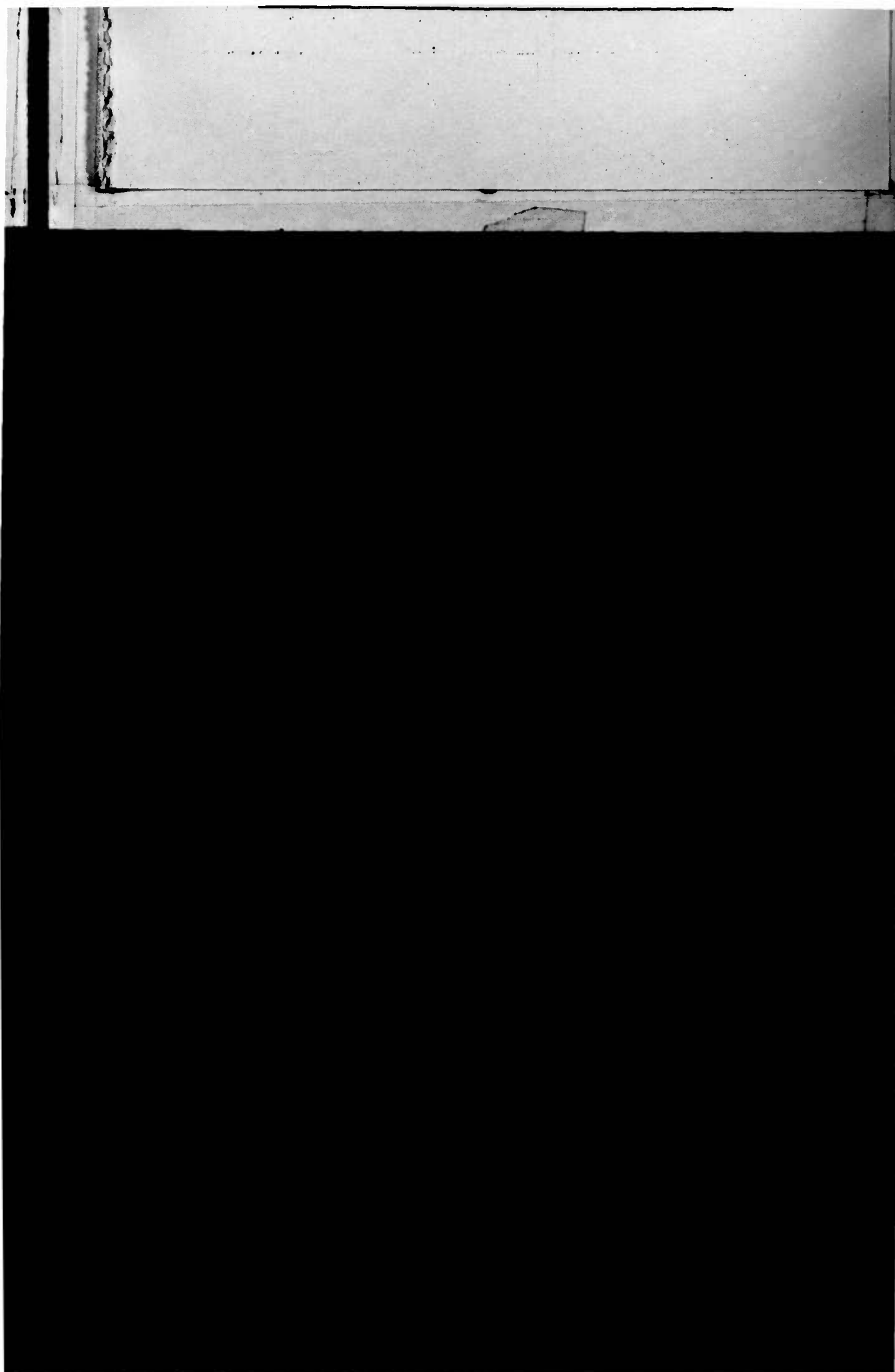
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